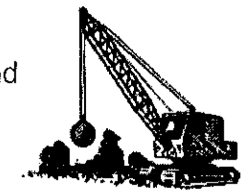
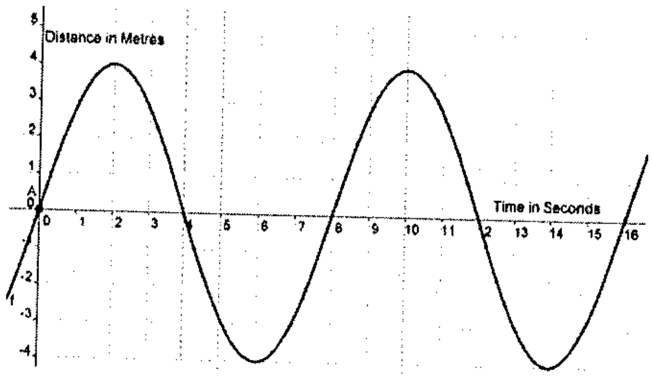
Unit 5 Lesson 4A | Application Problems

Question 1

A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and to the right of its resting position with respect to time is represented by the following graph.





a. What is the period of the crane's motion. Explain your answer?

b. What is the equation of the horizontal midway line of the curve and what does it represent?

c. What is the amplitude of the crane's motion? Draw a diagram to represent what the amplitude represents in terms of the motion of the ball?

d. How many complete swings will the wrecking ball make in four minutes?

e. What does point A represent?

Question 2

As a dolphin swims along by the side of a cruise ship he jumps to a height of four metres above the water surface and dives to a depth of four meters below the surface. He does this in a regular motion (simple harmonic motion). A passenger using a stopwatch and starting it just as the dolphin is at the surface determines that the dolphin completes one cycle every 8 seconds.

a. Describe the displacement of the dolphin relative to the water surface using a sinusoid curve and sketch the graph of the dolphin's displacement relative to the water surface

b After three seconds how high above the surface is the dolphin?

c Is the dolphin above or below the surface of the water after 37 seconds?

Question 3

The inside rim of a bicycle wheel whose diameter is 25 inches, is 3 inches off the ground. An ant is sitting on the inside rim of the wheel at the point 3 inches off the ground. Sean starts riding the bicycle at a steady rate. The wheel makes one revolution every 1.6 seconds.

a. Find the equation of a sinusoid curve which describes the motion of the ant and draw a graph of the function.

b What height in centimetres from the ground will the ant be 25 seconds into the trip given that 1 inch = 2.54 cm.?

c Within the first 10 seconds how many times will the ant be at its starting height?

Question 4

The number of people in thousands employed in a resort town is represented by the function

$$f(x) = 3.8 - 1.7cos\frac{\pi}{6}t$$

Take t = 0 as last day of January

Take t = 1 as last day of February

Take t = 2 as last day of March

a. Draw a rough sketch showing the variation in the number of people employed in the town for one complete period.

b When is the maximum number of people employed and what is this maximum number?

c During which months of the year will the number of people employed be 4,650 or greater?

d Does this model have any drawbacks and if yes identify one?

Question 5

A tsunami (tidal wave) is a fast moving wave caused by an underwater earthquake. The water oscillates about its normal level, with equal amplitudes above and below this level The period is fifteen minutes. Suppose that a tsunami with an amplitude of ten metres approaches the pier at Honolulu, where the normal depth of water is nine metres. Assuming that the depth of water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunami first reaches the pier.

a Two minutes, four minutes and twelve minutes.

b According to your model what will be the minimum depth of the water?

c How do you interpret this answer in terms of what will happen in the real world?

Question 6

A buoy in the ocean is bobbing up and down in harmonic motion.

At $t = 0$ seconds, the buoy is at its high point and returns to that high point every 8 seconds. The buoy moves a distance of 1.44 meters from its highest point to its lowest point.

a. Using a sinusoidal function create a mathematical model to represent the depth of water under the buoy and sketch the curve.

b How high is the buoy at time = 29 seconds?

c Is it rising or falling at that time?

d How many times in 2 minutes will the buoy be at sea level?

Unit 5 Lesson 4A | Homework

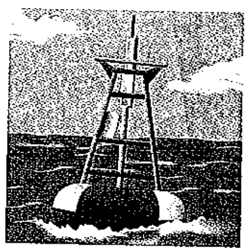
1. One particular July 4th in Galveston, TX, high tide occurred at 9:36 A.M. At that time the water at the end of the 61st Street Pier was 2.7 meters deep. Low tide occurred at 3:48 P.M., at which time the water was only 2.1 meters deep. Assume that the depth of the water is a sinusoidal function of time with a period of half a lunar day (about 12 hours 24 minutes).

   (a) At what time on the 4th of July did the first low tide occur?

   (b) What was the approximate depth of the water at 6:00 A.M. and at 3:00 P.M. that day?

   (c) What was the first time on July 4th when the water was 2.4 meters deep?

2. **Motion of a Buoy** A signal buoy in the Chesapeake Bay bobs up and down with the height h of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants a and b?



3. **Ferris Wheel** A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground, how long after reaching the low point is a rider 50 ft above the ground?

4. **Blood Pressure** The function

$$P = 120 + 30 \sin 2\pi t$$

models the blood pressure (in millimeters of mercury) for a person who has a (high) blood pressure of 150/90; t represents seconds.

(a) What is the period of this function?

(b) How many heartbeats are there each minute?

(c) Graph this function to model a 10-sec time interval.